known case. This gives further confidence to the use of the self-consistency of localization as a way of solving the localization problem in the relativistic case, in spite of the strange consequences that we then obtain.

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Relativistic Theory of the Elastic Dielectric

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1. The theories of elastic dielectrics proposed by Toupin (1963) and Grot and Eringen (1966) have been generalized by Salt (1969). The main assumptions and results of the latter work are summarized below.

2. A motion of a body can be represented by a set of three sufficiently smooth functions $X^K = X^K(x^i)$, which map a region of the space-time manifold, called the world tube of the motion, onto the body manifold B (Toupin, 1958). The $x^i, i = 1, 2, 3, 4$, are coordinates of the space-time point x, while the $X^K, K = 1, 2, 3$, are coordinates of the material point X in B. Space-time is a four-dimensional differentiable manifold, endowed with a fundamental tensor g_{ik} that has signature $(+,+,+,-)$. The functions $X^K(x^i)$ representing a motion satisfy the conditions that the matrix $\|\partial_i X^K\|$ be of rank 3, and that the three world vectors $\partial_i X^K$ be space-like.

The world velocity vector w^i is defined at each x in the world tube of the motion by the conditions

$$w_i w^i + 1 = 0$$

$$w^i \partial_i X^K = 0$$

3. At each point of the world tube of a motion an axial scalar mass density ρ may be defined such that mass is conserved:

$$\partial_i(\rho w^i) = 0$$

For any three-dimensional element of extension $d^3 v_i$ in space-time, $\rho w^i d^3 v^i$ is the mass of the element $d^3 X$ of B which is the image of $d^3 v_i$ under the mapping $X^K = X^K(x^i)$.

4. The change-current density j^i at each point in the world tube of a motion is determined by the magnetization-polarization tensor P^{ik} of the body at that point (Truesdell & Toupin, 1960):

$$j^i = \partial_k P^{ik}$$

Given the world velocity vector of the motion, P^{ik} can be expressed in terms of the world polarization and magnetization vectors P^i and M_i, (Møller, 1952). The necessary and sufficient condition that a material be non-magnetic is that M_i be identically zero.

5. The electromagnetic field can be represented by an anti-symmetric tensor field F_{ik} which is the curl of a vector potential A_i, $F_{ik} = 2\partial_{[i}A_{k]}$.

6. The field equations for the system consisting of an elastic dielectric interacting with a gravitational field and an electromagnetic field may be obtained from an action principle. The Lagrangian density L consists of the sum of four terms, representing (i) the gravitational field, (ii) the material, (iii) the electromagnetic field, and (iv) the interaction of the electromagnetic field with the material. These are

$$-(-g)^{1/2} R/2\kappa \qquad \text{(i)}$$

$$-\rho(1+\Sigma)(-g_{ik} w^i w^k)^{1/2} \qquad \text{(ii)}$$

$$-\tfrac{1}{4}(-g)^{1/2} g^{kl} g^{mn} F_{km} F_{ln} \qquad \text{(iii)}$$

$$\tfrac{1}{2} F_{ik} P^{ik} (-g_{mn} w^m w^n) \qquad \text{(iv)}$$

In (i) R is the scalar curvature of the space-time manifold and κ the gravitational constant. In (ii) Σ is an absolute scalar function of g_{ik}, $\partial_i X^K$, P^i and M_i; it might be interpreted as the internal energy per unit mass of the material.

The independent variables in the action principle are g_{ik}, X^K, ρ, w^i, A_i and P^{ik}. The following constraints have, however, to be taken into account by the use of undetermined Lagrange multipliers:

$$w_i w^i + 1 = 0$$

$$w^i \partial_i X^K = 0$$

$$\partial_i(\rho w^i) = 0$$

7. If the Lagrange multipliers are eliminated from the field equations that result from the above action principle, the following set of field equations is obtained:

$$(-g)^{1/2} G^{ik} + \kappa T^{ik} = 0$$

$$w^i \partial_i X^K = 0$$

$$\partial_i(\rho w^i) = 0$$

$$F_{ik} - 2\rho \frac{\partial \Sigma}{\partial P^{ik}} = 0$$

$$\partial_k P^{ik} - \partial_k[(-g)^{1/2} F^{ik}] = 0$$

G^{ik} is the Einstein tensor and T^{ik} the stress-energy-momentum tensor density, given by

$$T^{ik} = 2\frac{\delta L'}{\delta g_{ik}}$$

where L' is the sum of the last three terms in the Lagrangian density L, and $\delta/\delta g_{ik}$ denotes the Lagrange derivative.

8. In the non-relativistic approximation, the above field equations and the definition of T^{ik} reduce to the set of field equations and constitutive relations employed by Toupin (1963). Also, if the Lorentz invariant theory of Grot & Eringen (1966) is restricted to the case of zero heat conduction, their results may be shown to follow from the field equations in (7), provided the restriction to flat space-time is made and certain changes of the independent variables employed. Hence there is no inconsistency between the theories of Toupin and of Grot and Eringen.

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